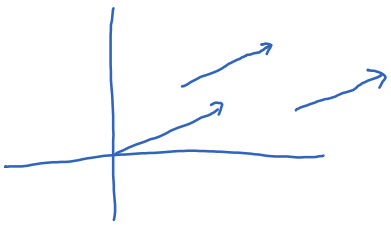
Lecture 3

Thursday, January 6, 2022 11:17 PM

* Prayer
* Spiritual thought

Vectors

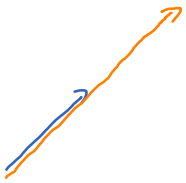


Geometrically, vector is represented by an arrow.

Algebraically, vector (in 2D) is a pair of numbers.

A vector has a dual nature:

Addition, scaling of vectors ....



length: $|v| = \sqrt{a^2+b^2}$

$v = \langle a, b \rangle$

Ex: $v = \langle 1, 2, 3 \rangle = i + 2j + 3k$

$w = \langle -2, 1, 1 \rangle = -2i + j + k$

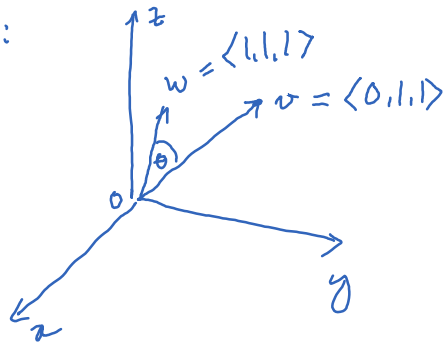
What is $v + 2w$?

How about the angle between two vectors?

$$\cos\theta = \frac{v \cdot w}{|v||w|} \qquad 0 \le \theta \le \pi$$

This formula provides a simple way to check if two vectors are perpendicular to each other.

Ex:



$$v \cdot w = \langle 0,1,1\rangle \cdot \langle 1,1,1\rangle = 2$$

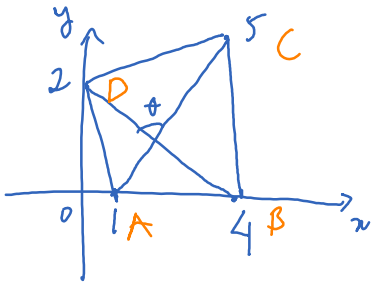
$$|v| = \sqrt{2}$$

$$|w| = \sqrt{3}$$

$$\cos\theta = \frac{2}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\theta \approx 0.355 \text{ (rad)} \approx 20.34^\circ$$

Ex



$A(1,0)$ $\qquad \vec{AC} = \langle 2,5\rangle$

$B(4,0)$ $\qquad \vec{BD} = \langle -4,2\rangle$

$C(3,5)$

$D(0,2)$

$$\cos\theta = \frac{\vec{AC} \cdot \vec{BD}}{|\vec{AC}||\vec{BD}|} = \frac{-8+10}{\sqrt{29}\sqrt{20}} = \frac{2}{\sqrt{580}} = \frac{1}{\sqrt{145}}$$

$$\theta \approx 85.24^\circ$$

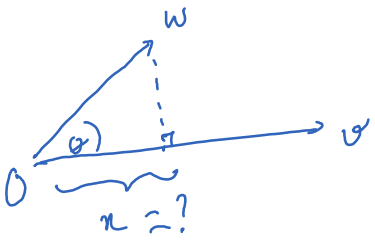
* Mathematica: $v = \{1, 2, 3\}$

$w = \{2, 1, 1\}$

$v.w$ / Norm[v] / Norm[w] $(= \cos\theta)$

ArcCos[%]

Projection:

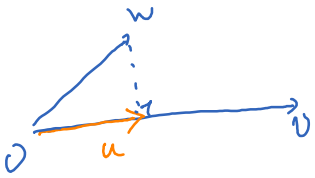


$x = |w| \cos\theta$

$= |w| \dfrac{v.w}{|v||w|} = \dfrac{v.w}{|v|}$

This is the scalar projection of w onto v.

$x = \text{comp}_v w$

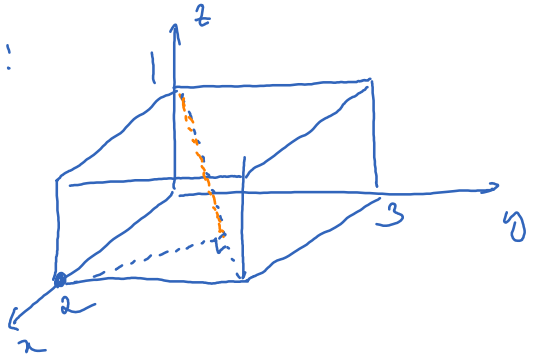


$u = x \dfrac{v}{|v|} = \dfrac{v.w}{|v|^2} v$

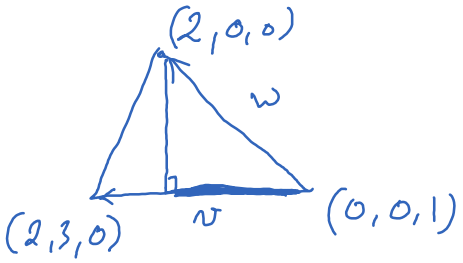
This is the vector projection of w onto v

$u = \text{proj}_v w$

Ex:

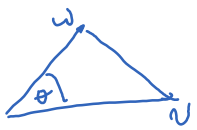


what is the length of the orange bar?



$w = \langle 2, 0, -1 \rangle$, $v = \langle 2, 3, -1 \rangle \rightsquigarrow$ $\text{comp}_v w = \dfrac{v.w}{|v|} = \dfrac{5}{\sqrt{14}}$

Cross product : useful tool to compute area of a triangle or parallelogram.



$v \times w$ is a vector

$v \cdot w$ is a number

$v \times w$ is a vector perpendicular to both v and w

$|v \times w| =$ area of parallelogram form by v and w

$v \times w$ is oriented by the right hand rule.